

7th Grade Expressions and Equations SOLUTIONS

1. The students in Mr. Sanchez's class are converting distances measured in miles to kilometers. To estimate the number of kilometers, Abby takes the number of miles, doubles it, then subtracts 20% of the result. Renato first divides the number of miles by 5, then multiplies the result by 8.

a. Write an algebraic expression for each method. **Let d represent the distance specified by Mr. Sanchez given in miles. Let k be the same distance expressed in kilometers. Then we have**

- **ABBY'S METHOD $k=2d-0.2d$**
- **RENATO'S METHOD $k=8(d/5)$**

b. Use your answer to part (a) to decide if the two methods give the same answer.

- **ABBY'S METHOD SIMPLIFIES TO $k=(2-.2)d=1.8d$**
- **RENATO'S METHOD SIMPLIFIES TO $k=(8/5)d=1.6d$**
THEY ARE NOT THE SAME!

c. Write an expression for the following sequence of operations: Add 3 to x , subtract the result from 1, then double what you have.

- **ADD 3 TO x TO GET $3+x$**
- **SUBTRACT THE RESULT FROM 1 TO GET $1-(3+x)$**
- **DOUBLE WHAT YOU HAVE TO GET**
 $2[1-(3+x)]$
THIS SIMPLIFIES TO
 $=2[1-3-x]=2[-2-x]=-4-2x$

d. Write an expression for the following sequence of operations: Add 3 to x , double what you have, then subtract 1 from the result.

- **ADD 3 TO x TO GET $3+x$**
- **DOUBLE WHAT YOU HAVE TO GET $2(3+x)$**
- **SUBTRACT 1 FROM THE RESULT TO GET**
 $2(3+x)-1$
THIS SIMPLIFIES TO
 $6+2x-1=5+2x$

2. Malia is at an amusement park. She bought 16 tickets, and each ride requires 2 tickets.

- a. Write an expression that gives the number of tickets Malia has left in terms of x , the number of rides she has already gone on. Find at least one other expression that is equivalent to it.

$$\begin{aligned}\text{Tickets Left} &= \text{Starting \# of Tickets} - \text{\# of Tickets Used} \\ &= 16 - (\text{\# of Rides})(\text{Ticket Cost per Ride}) \\ &= 16 - 2x\end{aligned}$$

- b. $16 - 2x$ represents the number of tickets Malia has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

(i) $14 - 2x$

Malia started with 14 tickets and this represents the number she has left when she has gone on x rides.

(ii) $16 - 4x$

Malia started with 16 tickets but she is with a friend so now every ride costs them together 4 tickets. Thus this is the number of tickets they have left when they have gone on x rides.

- c. $2(8 - x)$ also represents the number of tickets Malia has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

$2(8 - x)$ puts this expression in a form which allows us to see that the total possible number of rides is 8 because when $x = 8$ this is zero and if $x > 8$ then this expression would be negative which would not make much sense in this context. The factor of 2 is still essential for giving us the number of remaining tickets and not just the number of remaining rides.

(i) $2(7 - x)$

Malia can go on at most 7 Rides.

(ii) $4(4 - x)$

Malia and her friend can go on at most 4 rides.

3. If a woman making \$25 an hour gets a 10% raise, how much additional money will she make over the next day? Over the next week? Over the next month?

First we note that with her old rate she would make the following for various time periods of work

HOURS	PAY at \$25/hour
1	\$25
1 day= 8	25x8=\$200
1 week=5 days=40 hours	5x200=\$1000
1 month=4.5 weeks(on average)=180 hours	4.5x1000=\$4500

There are two equivalent ways to finish this calculation.

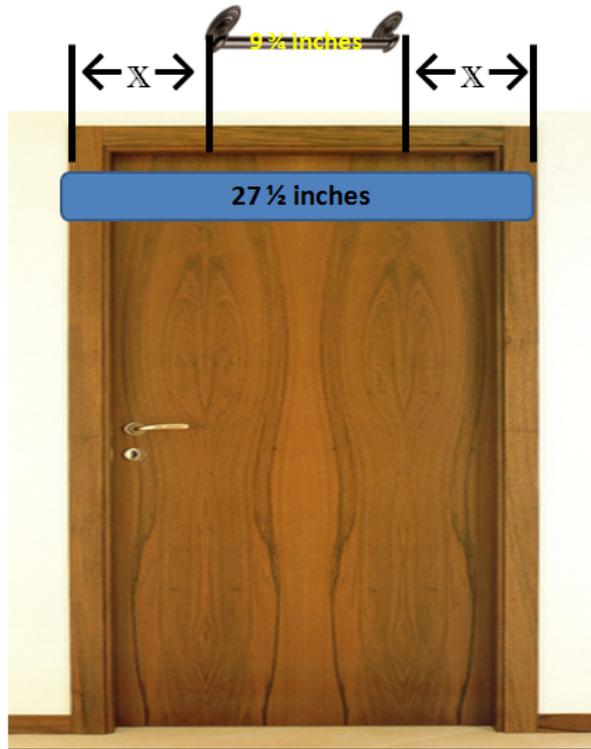
- We can note that her new hourly rate is $25+0.1 \times 25 = 25+2.50 = 27.50$ so now the table becomes

HOURS	PAY at \$27.50/hour	Increase in Pay
1	\$27.50	\$27.50-\$25=\$2.50
1 day= 8	27.50x8=\$220	\$220-\$200=\$20
1 week=5 days=40 hours	5x220=\$1100	\$1100-\$1000=\$100
1 month=4.5 weeks(on average)=180 hours	4.5x1100=\$4950	\$4950-\$4500=\$450

- OR We can note that all her old pay rates just need to be increased by 10%

OLD PAY	NEW PAY AT 10% MORE	Increase in Pay
1 Hour at \$25	25x1.1=\$27.50	\$2.50
1 day at \$200	200x1.1=\$220	\$20
1 week at \$1000	1000x1.1=\$1100	\$100
1 month at \$4500	4500x1.1=\$4950	\$450

4. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. What is the exact placement?



This diagram shows the power of algebra because we can see that x is the distance of the towel rack from either side of the door and

$$27\frac{1}{2} = 9\frac{3}{4} + 2x \text{ SO } 2x = 27\frac{1}{2} - 9\frac{3}{4} = \frac{55}{2} - \frac{39}{4} = \frac{110}{4} - \frac{39}{4}$$

$$= \frac{(110-39)}{4} = \frac{(111-40)}{4} = \frac{(171-100)}{4} = \frac{71}{4} = 17\frac{3}{4} = 2x$$

$$\text{Therefore, } x \text{ is } (17\frac{3}{4} \div 2) = (17 + \frac{3}{4}) \div 2 = (17 \div 2) + (\frac{3}{4} \div 2) = 8\frac{1}{2} + \frac{3}{8} = 8\frac{4}{8} + \frac{3}{8}$$

So $x = 8\frac{7}{8}$ inches. We should put the bar this far from the edge of the door.

(So in fact 9 inches is a very good estimate!)

5. Below is a table showing the number of hits and the number of times at bat for two Major League Baseball players during two different seasons: **(But expressed as a number between 0 and 1000)**

Season	Derek Jeter	David Justice
1995	12 hits in 48 at bats	104 hits in 411 at bats
1996	183 hits in 582 at bats	45 hits in 140 at bats

A player's *batting average* is the fraction of times at bat when the player gets a hit.

- a. For each season, find the players' batting averages. Who has the better batting average?

Season	Derek Jeter	David Justice	WHO is BETTER?
1995	$12/48=0.25$ so it's 250	$104/411\approx 0.25304$ so it's 253	JUSTICE
1996	$183/582\approx 0.3144$ so it's 314	$45/140\approx .3214$ so it's 321.	JUSTICE

- b. For the combined 1995 and 1996 seasons, find the players' batting averages. Who has the better batting average?

SEASON	JETER HITS	JETER AT BATS	JUSTICE HITS	JUSTICE AT BATS
1995	12	48	104	411
1996	183	582	45	140
COMBINED	$183+12=195$	$48+582=630$	$104+45=149$	$411+140=551$
AVERAGE		310		292

JETER is Better overall!

- c. Are the answers to (a) and (b) consistent? Explain.

It is indeed odd that Jeter wins overall and yet he doesn't win either of the individual seasons. However, Jeter's 314 in 1996 is far more significant in this calculation because he had relatively few at bats in 1995. Conversely, the Justice average of 321 in 1996 is not very significant since this was only based on 140 at bats versus the situation in 1995 where Justice had 411 at Bats.¹

¹ Notice that the fractions for 1995 and 1996 get combined by adding numerators and adding denominators – this is the naïve (but usually incorrect) fraction addition scheme!

6. Katie and Margarita have \$20.00 each to spend at Students' Choice book store, where all students receive a 20% discount. They both want to purchase a copy of the same book which normally sells for \$22.50 plus 10% sales tax.

- a. To check if she has enough to purchase the book, Katie takes 20% of \$22.50 and subtracts that amount from the normal price. She takes 10% of the discounted selling price and adds it back to find the purchase amount. Make this Calculation.

20% of \$22.50 is $0.2 \times 22.50 = \$4.50$

She subtracts this from the normal price $22.50 - 4.50 = \$18$

10% of the discounted price is $0.1 \times 18 = \$1.80$

Add this back to reduced price to give $\$18 + \$1.80 = \$19.80$

- b. Margarita takes 80% of the normal purchase price and then computes 110% of the reduced price. Make this calculation.

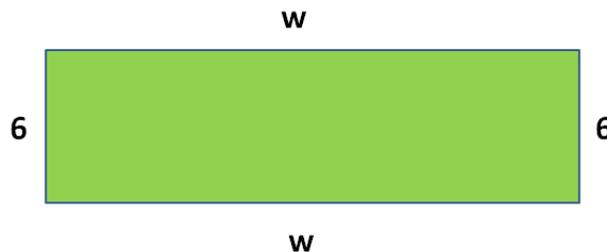
80% of the normal purchase price is $0.80 \times 22.50 = \$18$

110% of the reduced price is $1.10 \times 18 = \$19.80$

Is Katie correct? Is Margarita correct? Do they have enough money to purchase the book?

Both are correct and Margarita and Katie do have enough to purchase the books. Notice that we do indeed take discounts and then compute tax (i.e. in this order) because the tax is always computed last (i.e. at the time of purchase).

7. The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?



The perimeter is $6 + w + 6 + w = 54$ So $12 + 2w = 54$ So $2w = 42$ So $w = 21$. The width is 21 cm.

8. As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

Let P be the pay you earn in one week and let S be the number of sales you make. Then we have that

$$P=50+3S \text{ So we want } P=50+3S \geq 100$$

Solving this we get $3S \geq 100-50=50$. Thus $3S \geq 50$ and hence $S \geq 50/3=16 \frac{2}{3}$

We have to make at least 16 and $\frac{2}{3}$ sales. However, since we cannot make a fractional number of sales (they buy or they don't!) this means in reality we must make at least 17 sales.

9. Jonathan wants to save up enough money so that he can buy a new sports equipment set that includes a football, baseball, soccer ball, and basketball. This complete boxed set costs \$50. Jonathan has \$15 he saved from his birthday. In order to make more money, he plans to wash neighbors' windows. He plans to charge \$3 for each window he washes, and any extra money he makes beyond \$50 he can use to buy the additional accessories that go with the sports box set.

Write and solve an inequality that represents the number of windows Jonathan must wash in order to save at least the minimum amount he needs to buy the boxed set. Graph the solutions on the number line. What is a realistic number of windows for Jonathan to wash? How would that be reflected in the graph?

Let S be the total savings that Jonathan has and let W be the number of windows he washes. Then we have that

$$S = 15 + 3W$$

He needs at least \$50 so we have that

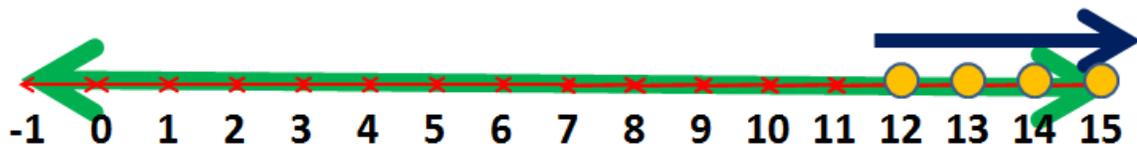
$$S = 15 + 3W \geq 50$$

So

$$3W \geq 35$$

So

$$W \geq 35/3 = 11 \frac{2}{3}$$



The blue indicates all possible solutions for W whereas the yellow dots show the real life solutions which must be integers greater than $11 \frac{2}{3}$. After all, if you only wash two thirds of a window then you will probably get paid nothing!