Grade 7 Ratios and Proportional Relationships SOLUTIONS

1. Suppose that Leo walks 1/2 mile in each 1/4 hour. At the same rate, how many miles will Leo walk in 1 hour? In 2 hours? How long will it take him to walk 1 mile? Two miles?

TIME (HOURS)	DISTANCE (MILES)
¼=15 minutes	½
2/4=1/2=30 minutes	1
¾=45 minutes	1 ½
1=60 minutes	2
2	4

In 1 hour Leo can walk 2 miles. If he has the endurance then in 2 hours he can walk 4 miles. 1 mile will take him 30 minutes or ½ hour and 2 miles will take 60 minutes or 1 hour. Notice that the unit rate is ½ / ¼ =4/2=2 miles per hour.

2. Travis was attempting to make muffins to take to a neighbor that had just moved in down the street. The recipe that he was working with required 24 cups of sugar, 12 cups of butter, 1 pound of flour, and 16 cups of blueberries.

Travis accidentally put an extra 3 whole cups of butter in the mix.

(a) What is the ratio of sugar to butter in the original recipe? What amount of sugar does Travis need to put into the mix to have the same ratio of sugar to butter that the original recipe calls for?

In the original recipe we have

$= \frac{\text{12 cups of butter}}{\text{24 cups of sugar}} = 1/2$ cup butter per cup sugar

If we added 3 cups of butter then we need to add 3x2=6 cups of sugar.

(b) If Travis wants to keep the ratios the same as they are in the original recipe, how will the amounts of all the other ingredients for this new mixture compare to the amounts for a single batch of muffins?

In the original recipe we have

12 cups of butter / 1 pound of flour or 1/12 flour(lbs) to butter (cups). Thus we need an additional

(1/12)(pounds/cup) x 3cups = (3/12) pounds= ¼ pounds of flour.

In the original recipe we have 16 cups of blueberries/12 cups of butter

=(16/12) blueberries(cups) per butter (cup)=(4/3) blueberries/butter.

Thus if we add an additional 3 cups of butter then we will need an additional

(4/3) x 3 = 4 cups of blueberries.

This got Travis wondering how he could remedy similar mistakes if he were to dump in a several cups of some of the other ingredients. Assume he wants to keep the ratios the same.

(c) How many cups of sugar are needed if a single cup of blueberries is used in the mix?

The original recipe calls for 24 cups of sugar per 16 cups of blueberries

=(24/16) sugar (cups) per blueberries (cups) = (3/2) sugar per blueberry. So if the recipe calls for 1 cup of blueberries we will need

(3/2) sugar/blueberry x 1 blueberry= (3/2) cups of sugar = 1 ½ cups of sugar.

(d) How many cups of butter are needed if a single cup of sugar is used in the mix?

The original recipe calls for 12 cups butter/24 cups of sugar

=(12/24) butter (cups) per sugar (cups)= ½ butter per sugar . Thus a single cup of sugar requires

(½ butter /sugar) x 1 sugar = ½ cup of butter.

(e) How many cups of blueberries are needed for each cup of sugar?

Above we found 3/2 sugar cup per blueberry cup so this means we have 2/3 blueberry cup per sugar cup (i.e. the reciprocal). Therefore, this means if we add an additional cup of sugar then we need

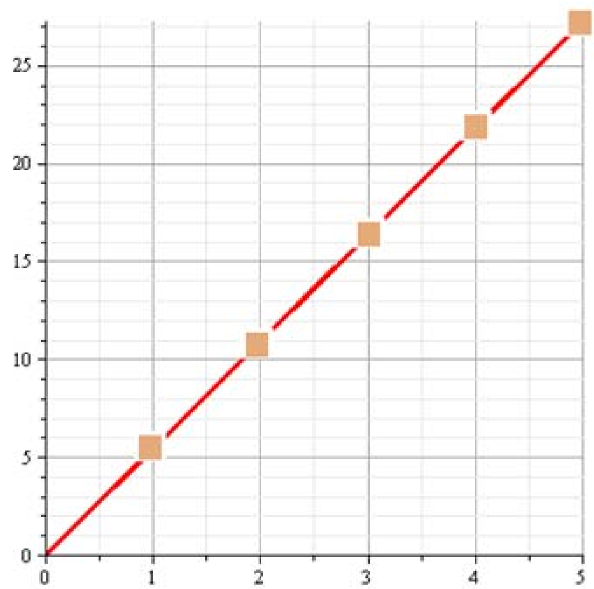
(2/3) blueberry/sugar x 1 sugar= 2/3 cup of blueberries.

3. Coffee costs $16.35 for 3 pounds.

(a) What is the cost for one pound of coffee?

The cost ratio is (16.35/3) $ per pound=$5.45 per pound. Thus one pound costs $5.45.

(b) At this store, the price for a pound of coffee is the same no matter how many pounds you buy. Let x be the number of pounds of coffee and y be the total cost of x pounds. Draw a graph of the relationship between the number of pounds of coffee and the total cost. **$y=5.45\,x$**



Pounds of Coffee	$ COST
1	$5.45
2	$10.90
3	$16.35
4	$21.80
5	$27.25

(c) Where can you see the cost per pound of coffee in the graph? What is it? **The cost per pound is the slope of the graph.**

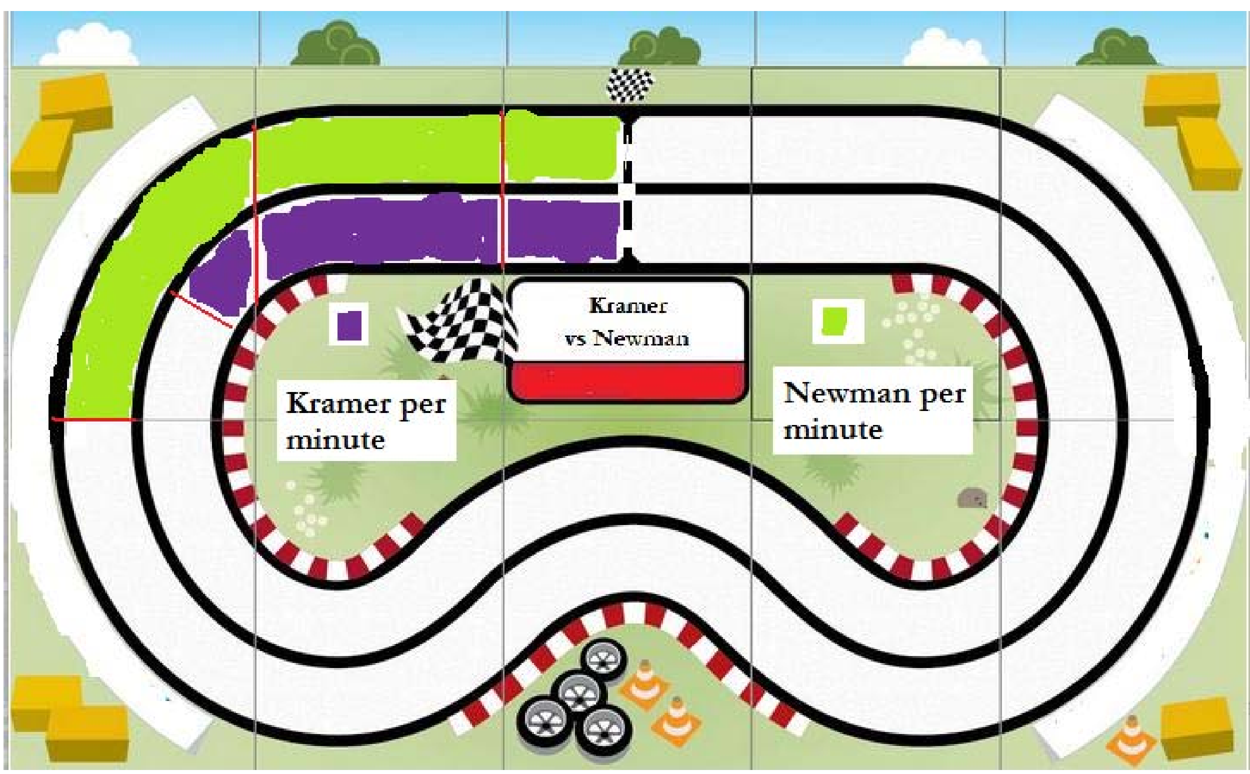
4. 2 Bicyclists practice on the same course. Kramer does the course in 6 minutes and Newman does it in 4 minutes. They agree to race each other 5 times around the course. If Kramer has a 4 minute head start, then how soon after the start will Newman overtake Kramer?

Kramer's rate is 6 minutes per lap or 1/6 th of a lap per minute.

Newman's rate is 4 minutes per lap or 1/4th of a lap per minute.

In the diagram we have shown how far each of the racers can travel in one minute. So if we measure time from when Newman starts then we have the following minute by minute description of the start of the race. As we see, Newman catches up after 8 minutes when they have both completed 2 laps

TIME (MINUTES)	KRAMER (LAPS)	NEWMAN (LAPS)
0	4/6	0
1	5/6	¼
2	1	2/4
3	1 1/6	¾
4	1 2/6	1
5	1 3/6	1 ¼
6	1 4/6	1 2/4
7	1 5/6	1 ¾
8	2	2



4. A text book has the following definition for two quantities to be directly proportional:

We say that y is directly proportional to x if $y=kx$ for some constant k.

For homework, students were asked to restate the definition in their own words and to give an example for the concept. Below are some of their answers. Discuss each statement and example. Translate the statements and examples into equations to help you decide if they are correct.

- Marcus:

  This means that both quantities are the same. When one increases the other increases by the same amount. An example of this would be the amount of air in a balloon and the volume of a balloon.

  Two quantities could be directly proportional wit k=1 so y=x but that is a more specialized kind of proportional relationship. The example Marcus gives is a case where the amount of air and volume are indeed the same but they don't have to be this way. This would be such a restriction on the definition that the concept would no longer be very useful. In fact, two quantities measured in entirely different units (like dollars and pounds of coffee) can be directly proportional and we cannot have k=1 here since these are not directly comparable quantities.

- Sadie:

  Two quantities are proportional if one change is accompanied by a change in the other. For example the radius of a circle is proportional to the area.

  Katie's example illustrates two quantities which are certainly related but they are not directly proportional since $A=\pi r^2$. A and r^2 are directly proportional by not A and r. Direct proportionality is a specialized kind of relationship between quantities. To see this note that as r goes from 0 to 1, A goes from 0 to π. If they were directly proportional then we would need that as r goes from 1 to 2 A would go from π to π+ π=2 π. However, that is not what happens – A goes from π to 2^2 π=4 π.

- Ben:

  When two quantities are directly proportional it means that if one quantity goes up by a certain percentage, the other quantity goes up by the same percentage as well. An example could be as gas prices go up in cost, food prices go up in cost.

  This is a poor explanation because percentage changes have to relate to a whole whereas absolute changes do not.

  If two quantities x and y both start from zero and they are directly proportional (i.e. y=kx) then for the most part a percentage change in one will lead to an equal percentage change in the other. To see this note that an increase of r% for x means x gets replaced by (1+r)x so y is replaced by (1+r)y=k(1+r)x which is equivalent to our original equation. However, when x changes from say 0 to 1, we cannot assign a percentage change to x and so the percentage change idea looses its usefulness.

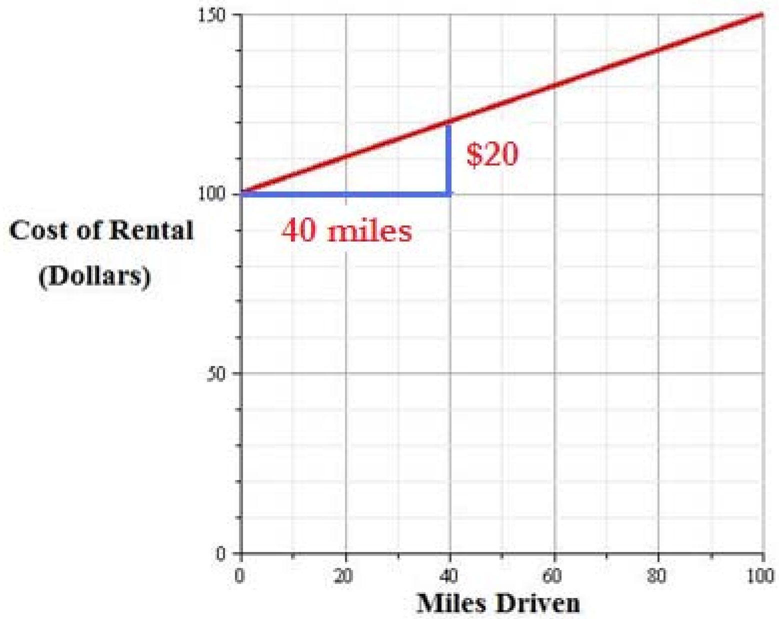
  In addition, we sometimes need to consider situations where the <u>changes</u> to two quantities are proportional so percentage changes would not necessarily be. For example, imagine that the population of Reno is 300,000 and the population of Sparks is 400,000 people. Further let us suppose that for every 100 people moving to Reno there are 200 who move to Sparks. Thus the absolute increases in population for Sparks (call this S) and the absolute increase in population for Reno (call this R) are related by S=2R. However, percentage increases in the total population (to talk about percentages you need to reference a whole) are not the same – if the Reno population doubles (a 100% increase) to 600,000 then the Sparks population will go up by 2x300,000=600,000 to a total of 1 million and this is an increase of 600000/400000=150%.

  Finally, confusing percentages and time with linear relationships and time can be quite wrong. For example, if my savings grows at 10% per year then I would go from $100 to $110 in the first year (an increase of $10) but from $110 to $121 (an increase of $11) in the second year. This represents a total increase of $21 (i.e. 21%) in 2 years.

  Jessica:

  When two quantities are proportional, it means that as one quantity increases the other will also increase and the ratio of the quantities is the same for all values. An example could be the circumference of a circle and its diameter, the ratio of the values would equal π

THIS IS CORRECT!



5. The above is a graph showing the cost of a rental car based on the number of miles driven. There is a basic cost for renting the car and then a per mile charge for the miles driven. What is the cost per mile? How does the graph tell you this? What is the basic cost of the rental? Where do you see this on the graph? Is the cost of the rental proportional to the number of miles driven?

The cost per mile is the slope of the graph which is (20/40) $ per mile=$0.50 per mile. The basic cost of the rental is what you would pay if you didn't drive any miles – this is the y-intercept of the graph which is $100.

The cost of the rental is <u>not</u> proportional to the number of miles driven because we d not have Cost=k x Miles but rather we have Cost=k x Miles+100.(i.e. if two quantities are proportional then when one is zero the other must also be zero).

6. There are 270 students at Colfax Middle School, where the ratio of boys to girls is 5:4. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is 4:5. The two schools hold a dance and all students from both schools attend. What is the ratio of boys to girls at the dance?

For Colfax we must relate the desired part to part ratio to a part to whole ratio. Thus from 5:4 we note that 5+4=9 and 270÷9=30. Thus we have 30 x 5=150 boys and 30x4=120 girls. For Winthrop we have the same divisor (4:5 goes to 4+5=9) but a different total so we consider 180÷9=20. Thus there are 4x20=80 boys at Winthrop and 5x20=100 girls at Winthrop.

SCHOOL	BOYS	GIRLS	Ratio B:G
COLFAX	150	120	5:4
WINTHROP	80	100	4:5
COMBINED	230	220	23:22

Notice we cannot add the ratios – we ave to combine absolute quantities and then look at the ratio!

7. After eating at your favorite restaurant, you know that the bill before tax is $52.60 and that the sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much should you leave for the waiter? How much will the total bill be, including tax and tip? Show work to support your answers.

To compute the tip we take the before tax amount and compute 20%

0.20 x $52.60 = $10.52.

To compute the tax we take the pre-tip amount and add 8%

0.08x$52.60=$4.208. To the nearest penny this is $4.21

The total bill is thus

$52.60 + $4.21 + $10.52 = $67.33