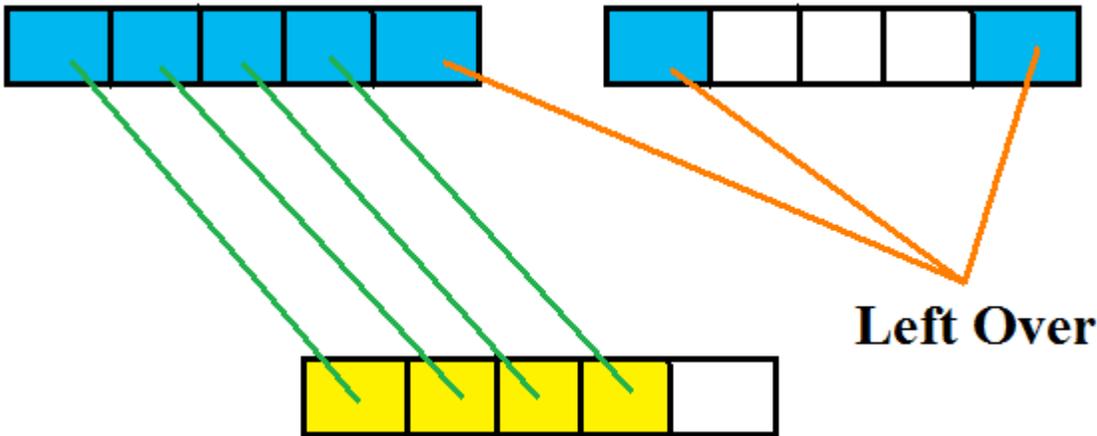


NMP Teachers' Pre-Test 2014 **SAMPLE SOLUTIONS**

1. Write a story problem for  $1\frac{2}{5} \div \frac{4}{5}$ , answer the problem you created, then use an area model to illustrate the operation. Explain your remainder and how that relates to your diagram as well.

*Sammy has a special ribbon that is one and 2/5ths feet long. He wants to use this to make special art gifts for his family. Each art gift requires 4/5ths of a foot of ribbon. How many art gifts can he make and what does he do with what is left over?*

**SOLUTION:**



*We see that the original Ribbon available (in blue) is enough for one special gift (which requires the ribbon shown in yellow). We then have 3/5ths feet left over and this is 3 of 4 chunks needed for another gift. i.e. to make the second gift we need just one additional 1/5<sup>th</sup> of a foot of ribbon (i.e. a 4<sup>th</sup> of a project) So the answer is 1 and 3/4ths.*

2. David bought a shirt on sale that was 20% less than the original price.
  - a) If the original price was \$10 more than the sales price, what was the original price?  
*So \$10 represents 20% of the original price i.e. 1/5<sup>th</sup> of the original price. Thus the original price must have been  $5 \times \$10 = \$50$ .*
  - b) If the sales price was \$100, how much was the original price?  
*Now we have that \$100 is 80% or 4/5ths of the original price. Dividing \$100 into 4 equal parts gives \$25 for each part. Thus each 1/5<sup>th</sup> of the original price is \$25 so the original price is  $\$100 + \$25 = \$125$ .*

3. At Camp Wigwam there are 150 campers with a 3 to 2 ratio of boys to girls. At Camp Richardson there is a 2:3 ratio of boys to girls and there are 40 boys.
- a) The two camps are getting together for a dance. What will the ratio of boys to girls be at the dance?

*For Camp Wigwam we split the 150 kids into 5 equal parts of 30 kids each. Three of these (i.e. 90) are boys and two of these (i.e. 60) are girls.*

*At camp Richardson the 40 boys are 2 of 5 parts so each part is 20 boys. Three of these parts is 60 girls.*

<b>CAMP</b>	<b>BOYS</b>	<b>GIRLS</b>
<b>Wigwam</b>	90	60
<b>Richardson</b>	40	60
<b>TOTAL</b>	<b>130</b>	<b>120</b>

*So at the Dance will have a ratio for boys to girls of 13:12.*

- b) At another camp the ratio of boys to girls is 3:1. If 4 more girls join the camp the ratio changes to 3:2. How many children are now at this camp?

*Let  $x$  be the number of boys at the camp originally. Thus the original number of girls is  $x/3$ . Now with the revised numbers we have  $x/3+4$  girls. With the revised ratio we see that the number of boys is 3 halves of this or  $3(x/3+4)/2$ . Since the number of boys has not changed we must have*

$$3(x/3+4)/2=x$$

$$\text{So } 3(x/3+4)=2x$$

$$\text{So } x+12=2x$$

$$\text{So } x=12.$$

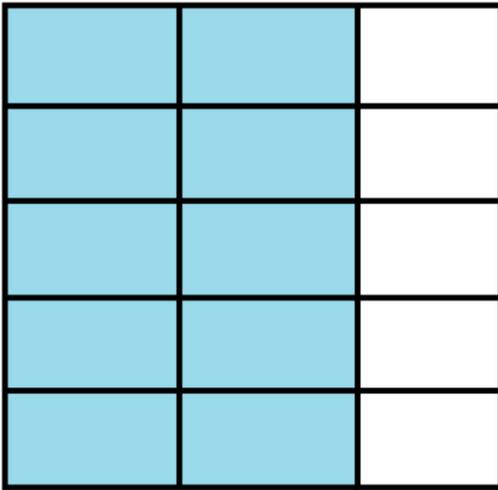
*Therefore the original camp had 12 boys and  $12/3=4$  girls.  
This changed to 12 boys and  $4+4=8$  girls for a final total of 20 kids.*

4. Write a story problem for  $\frac{2}{3} \times \frac{4}{5}$ , answer the problem you created, then use an area model to illustrate the operation.

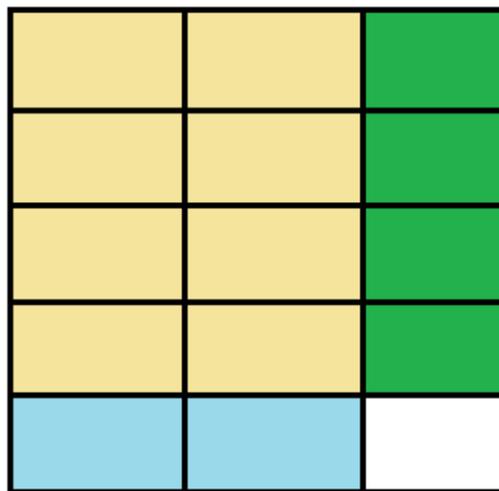
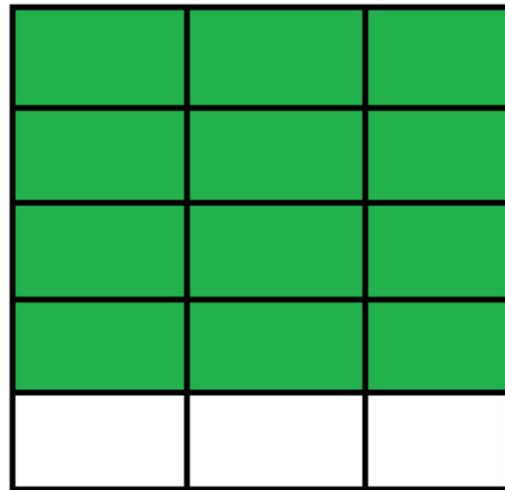
*Richie Rich is considering the purchase of Teruni Corp for the listed price of  $\frac{2}{3}$  of a billion dollars. After crunching the numbers he decides that he is only willing to pay 80% or  $\frac{4}{5}$  of the list price. What is the price that Richie Rich is willing to pay for Teruni Corp?*

**SOLUTION:** *He pays  $\frac{8}{15}$  of a billion dollars for Teruni Corp.*

**$\frac{2}{3}$  of a Billion Dollars**



**$\frac{4}{5}$  of a whole**



**Overlap is 8 equal sized rectangles of a total of 15 equal sized rectangles for the whole**

5. Cathy works two jobs. At Learners she earns \$18 per hour and 10% of each sale she makes.
- Write an expression that shows how much Cathy earns over time.  
*Let  $S$  be the total dollar amount of sales Cathy makes at Learners and let  $H$  be the number of hours she works. Then she will make  $18H+0.1S$*
  - At Seagles she earns \$12 and hour and \$5 for each sale. Write an expression for her sales at Seagles. Explain the similarities and/or differences between the two expressions, Learner's and Seagles.  
*Let  $N$  be the total number of sales Cathy makes at Seagles and let  $T$  be the total number of Hours she works there. Then she will make  $12T+5N$ .*

*With the information given it is impossible to say which is the better paying environment. Suppose that in one hour Cathy sells 20 Gumballs for 5cents each. At Seagles she would make  $\$12+\$5(20)=\$112$ . At Learner's she would make (with total sales of \$1.00)  $\$18+0.1(\$1.00)=\$18.10$ .*

*In contrast if she took 1 hour to sell one Mazerati for \$100,000 then at Learners she would make  $\$18+0.1(\$100,000)=\$10,018$  whereas at Seagles she would only bring in  $\$12+\$5(1)=\$17$ .*

6. Explain how you would use Mental Math to solve each of the following.
- $(24 * 8) + (6 * 8) = (24+6)*8 = 30*8 = 240$
  - $53 - 27 = 56 - 30 = 26$
  - $40 * 99 = 40*(100-1) = 40*100 - 40 = 4000 - 40 = 3960$ .
  - $235 + 123 = 235 + 100 + 20 + 3 = 335 + 20 + 3 = 355 + 3 = 358$
  - $220 \div 5 = (220 * 2) \div (5 * 2) = 440 \div 10 = 44$

7. Southwest Baking is creating a tortilla that is made of a blend of corn and rice flour. The company is not disclosing the percentage of each ingredient in the blend, but we know that the corn in the blend contains 9% protein, and the rice flour in the blend contains 6% protein. Overall, each 100 gram serving of tortilla contains 7.8 grams of protein. How much corn and how much flour is in one serving of the tortillas?

*Let  $x$  be the percentage of Corn in the mix and  $y$  the percentage of Rice in the mix. Thus the 100 grams of mix is comprised from  $x$  grams of Corn and  $y$  grams of Rice. Also, in 100 grams of the mix we have  $0.09x$  grams of protein (from the Corn) and  $0.06y$  grams of protein (from the Rice). This makes for a total of 7.8 grams of Protein*

$$x+y=100 \text{ or } 9x+9y=900$$

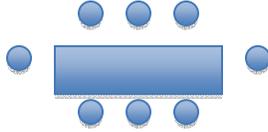
$$0.09x+0.06y=7.8 \text{ or } 9x+6y=780$$

*Subtracting the second equation from the first equation gives us*

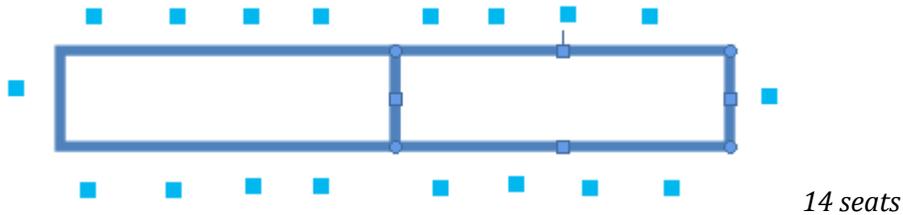
$$(9x-9x)+(9y-6y)=900-780 \text{ or } 3y=120$$

*Thus  $y=40$  (Grams of Rice) and  $x=60$  (Grams of Corn) are in the mix.*

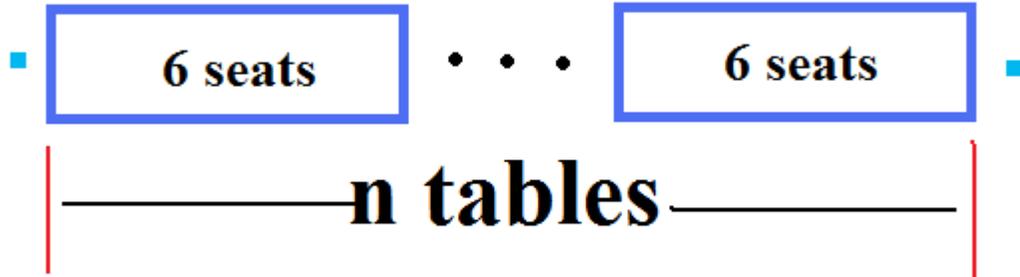
8. Tables for a wedding are being set up outside. They sit 8 people as shown below.



- a) How many people will 2 tables seat if the shorter ends are butted together.



- b) Create an expression for how many people  $n$  tables will seat.



$6n+2$  people can be seated at the  $n$  tables (each new table gives 6 more seats) but the two tables on the end always give 2 more seats as long as we have  $n \geq 2$ .

9. Jeff had one-fourth as much money as Peggy. Ed had twice as much money as Peggy. They counted out their money and then gave \$20 to one of their friends. If they now have a total of \$84, how much money did each initially have? Write an equation for this problem and solve it.

*They originally had  $\$84 + \$20 = \$104$ . Ed has 4 times as much as Jeff and twice as much as Peggy so in total that is 1 (for Jeff) + 4 (for Peggy) + 8 (for Ed) = 13 parts to the money.  $\$104$  divided by 13 is 8. Thus we have that*

*Jeff had \$8.*

*Peggy had four times this much or \$32.*

*Ed had twice this much or \$64. (Note that  $8 + 32 + 64 = 104$ )*

10. Tom and Lucy had both just run out of money when they got to start their summer jobs. For every 3 hours Tom works, he earns \$27. Lucy's earnings are listed below.

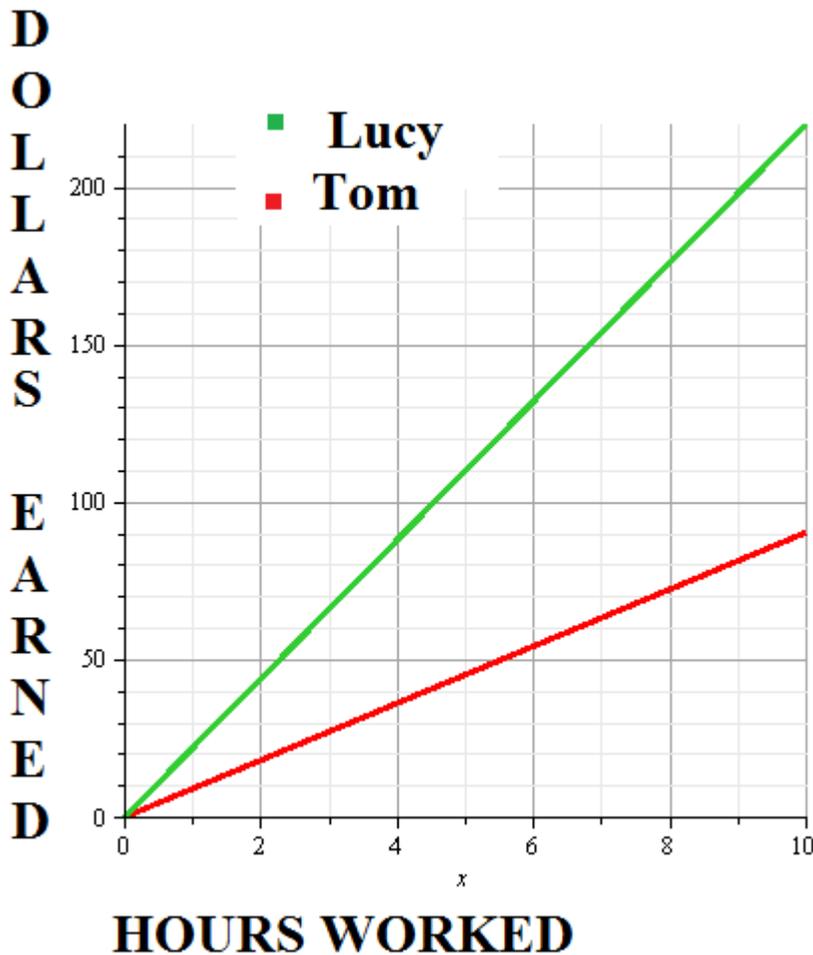
Lucy's Earnings

Time	Total Earnings
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1	22
2	44
3	66
4	88

- a) Create equations for both Tom and Lucy's earnings given how many hours they have worked, and graph them.

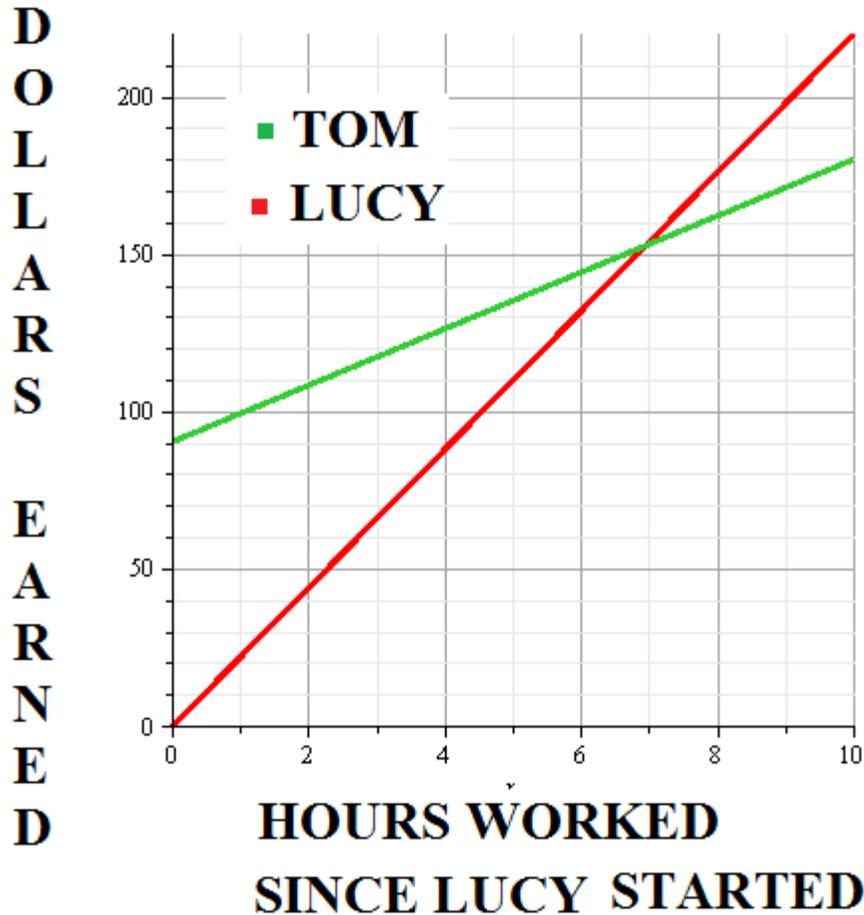
*Tom earns  $\$27/3=\$9$  per hour and Lucy earns  $\$22$  per hour (Total earnings in her table equals  $22 \cdot \text{time}$ ). Thus for Tom we have  $T=9h$  and for Lucy we have  $L=22h$ . Here  $h$  represents the number of hours worked and  $T$  and  $L$  are the earnings of Tom and Lucy respectively.*



- b) If Tom started working 10 hours before Lucy did, how long will it be until they have made the same amount of money? Show this algebraically as well as graphically.

*Before Lucy starts Tom will have earned  $\$9(10)=\$90$ . Let  $h$  be the number of hours of work starting when Lucy begins. Thus we have*

$90+9h=22h$  so  $13h=90$  and this  $h=90/13$  hours or equivalently 6 and  $12/13$  hours. (Will they pay for partial hours worked?) In the graph below we want the point where the lines  $90+9h$  and  $22h$  intersect. Notice the colors are reversed from above and this is a different interpretation of  $h$ .



- c) What is the constant of proportionality in Tom's equation and what does it mean in terms of earnings and time? Where is this found on the graph?

*These are the slopes of the lines. For Tom this is \$9 dollars per hour and for Lucy this is \$22 per hour. Slope always has units of the y-axis (in this case dollars) divided by the units of the x-axis (in this case hours).*